Collective Shape-Phases of Interacting Fermion Systems

Yu-xin Liu,[1,2] Zhan-feng Hou,[1] Yu Zhang,[1] and Haiqing Wei[3]

[1]*Department of Physics and the CMOE Key Laboratory of Heavy Ion Physics, Peking University, Beijing 100871, China*
[2]*Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China*
[3]*School of Information Science and Engineering, Lanzhou University, Lanzhou 730000, China*

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A microscopic theory is presented for identifying shape-phase structures and transitions in interacting fermion systems. The method provides a microscopic description for collective shape-phases, and reveals detailed dependence of such shape-phases on microscopic interaction strengths. The theory is generally applicable to fermion systems such as nuclei, quarks, and in particular trapped cold atoms, where shape-phases may be observed and investigated in a controlled manner.

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Collective motion is common in fermion systems such as nuclei and molecules. Nuclei have been found for a long time to possess interesting geometric shapes, such as a vibrating spheroid, an ellipsoid, and exotically deformed shapes [1, 2]. Pairing induced deformations (PIDs) have recently been observed in trapped fermion systems of cold atoms [3, 4]. Mean-field theories have been used to describe the properties of such fermion systems as many-electron atoms, nuclei, quarks, and trapped atomic fermions [5, 6, 7]. A long-standing question is to identify the shape phase structures and transitions in the general framework of mean-field theory in fermion space. Only recently there have been studies on nuclear shape-phase transitions [8, 9] and critical point symmetries [10] in the framework of shell model having either monopole-pairing and quadrupole-quadrupole (QQ) interactions or monopole-pairing and quadrupole-pairing and on the shape coexistence and evolution in superheavy nuclei in the energy density functional theory involving monopole-pairing term [2]. But the overall structure of nuclear shape-phases is still unclear, especially in a more general mean-field model that incorporates not only monopole-pairing and QQ interactions but also one-body terms and quadrupole-pairing. In similar systems of trapped atoms, there is also a lack of theoretical understanding of PIDs observed in recent experiments [3, 4].

Starting from a microscopic theory of interacting fermion systems, this paper presents a systematic method for identifying shape-phase structures and transitions in such systems, so to provide a unified microscopic description for collective shape-phases of both nuclei and trapped cold atoms, as well as other similar systems. From a microscopic viewpoint, our method recovers the rich structures and transitions of the shape-phases (dynamical symmetries) that have been experimentally observed and accounted for by the interacting boson model (IBM) [11, 12]. More importantly, our method reveals detailed dependence of such shape-phases on the interaction strengths in a mean-field Hamiltonian. The same theory may be applied to other systems such as trapped cold atoms with tunable interaction parameters, therefore points to the possibility of observing and controlling shape-phases and transitions therein.

As an approximation of the shell model, a successful mean-field theory of nuclear structure, the IBM has been proven highly successful in describing low-lying collective features of medium and heavy mass nuclei [12, 13]. In particular, it has well accounted for nuclear shape-phase structures and transitions [11, 12, 14, 15, 16, 17, 18, 19, 20]). The success can be attributed to the incorporation of collective degrees of freedom and the simplification of calculations by a mapping from fermions to bosons. Three dynamical symmetries, U(5), SU(3) and O(6), are naturally incorporated into the IBM, which correspond respectively to shape-phases of a spheroid, axially prolate rotor and γ-soft rotors [11, 12]. There is also $\overline{\mathrm{SU}(3)}$ symmetry corresponding to an axially oblate rotor phase[12]. A widely used mean-field Hamiltonian in fermion space can be written as[21]

$$\hat{H}_F = \sum_{jm} \varepsilon_j a^\dagger_{jm} a_{jm} - \frac{1}{2} g_0 \hat{P}^\dagger_0 \hat{P}_0 - \frac{1}{2} g_2 \hat{P}^\dagger_2 \cdot \hat{P}_2 - \frac{1}{2} k :\hat{Q}_2 \cdot \hat{Q}_2: , \tag{1}$$

with

$$\hat{P}^\dagger_0 = \sum_{jm} a^\dagger_{jm} \tilde{a}^\dagger_{jm} , \tag{2}$$

$$\hat{P}^\dagger_{2\mu} = \sum_{j_1 m_1 j_2 m_2} < j_1 m_1 | q_{2\mu} | j_2 m_2 > a^\dagger_{j_1 m_1} \tilde{a}^\dagger_{j_2 m_2} , \tag{3}$$

$$\hat{Q}_{2\mu} = \sum_{j_1 m_1 j_2 m_2} < j_1 m_1 | q_{2\mu} | j_2 m_2 > a^\dagger_{j_1 m_1} a_{j_2 m_2} , \tag{4}$$

where ε_j is the single-particle energy, $: :$ denotes the normal product of fermion operators, g_0, g_2, k are strengths of monopole-pairing, quadrupole-pairing, and QQ interaction, respectively, $\tilde{a}_{jm} = (-1)^{j-m} a_{j-m}$ and $q_{2\mu} = r^2 Y_{2\mu}$. Besides the energy spectrum, a particularly interested property is the electric quadrupole transition rate, $B(E2, L_i \to L_f) = \frac{1}{2L_i+1} \langle L_f || \hat{T}(E2) || L_i \rangle^2$, with $\hat{T}(E2)$ being $\hat{Q}_2$ multiplied by an effective charge.

We take a nuclear system as a numerical example, although the same theory and numerical procedure can be applied to other fermion systems equally well. To simplify the calculations, the method of Dyson-type boson mapping [22, 23] is employed with the SD pair truncation keeping only pairs with angular momentum $J = 0$ and $J = 2$ [24]. Then the Hamiltonian takes an IBM form $\hat{H}_B(S^\dagger, S, D^\dagger, D)$, with the S, D pairs being regarded as s, d bosons, respectively. Protons and neutrons are not differentiated, so the model is actually corresponding to the IBM-1. The single-particle wave functions are chosen to be harmonic oscillator's with oscillation constant $b^2 = 1.0A^{1/3}fm^2$, $A = 130$. For the 50-82 shells, the active single-particle orbits are $2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 1h_{11/2}, 2d_{3/2}$ with energies $0, 0.8, 1.3, 2.5, 2.8$ MeV, respectively, similar to those used in Ref. [25]. The final truncated Hamiltonian is diagonalized in the U(5)-symmetric bases with total number of nucleon pairs being set to $N = 10$.

To identify the shape-phases, it is helpful to examine the correspondence between the interaction strengthes in the microscopic model and the dynamical symmetries in the IBM. Quantities of interest are the energy ratios $R_{42} = \frac{E_{4_1}}{E_{2_1}}$, $R_{62} = \frac{E_{6_1}}{E_{2_1}}$, $R_{02} = \frac{E_{0_2}}{E_{2_1}}$, and $R_{22} = \frac{E_{2_2}}{E_{2_1}}$, with $E_{0_1} = 0$, and ratios of the electric quadrupole transition rates $B_{42} = \frac{B(E2;4_1\to 2_1)}{B(E2;2_1\to 0_1)}$, $B_{64} = \frac{B(E2;6_1\to 4_1)}{B(E2;2_1\to 0_1)}$, $B_{02} = \frac{B(E2;0_2\to 2_1)}{B(E2;2_1\to 0_1)}$, and $B_{22} = \frac{B(E2;2_2\to 2_1)}{B(E2;2_1\to 0_1)}$, which are known to be able to characterize the low-lying energy spectrum well. Table I lists values of these quantities in the dynamical symmetries of IBM-1 (with total boson number $N = 10$), which are to be compared with those obtained in the microscopic model.

TABLE I: Values of interested quantities of a 10-boson system in the IBM. Those marked with a star depend on additional parameters in the Hamiltonian (as given in Refs. [16, 19]).

	R_{42}	R_{62}	R_{02}	R_{22}	B_{42}	B_{64}	B_{02}	B_{22}
U(5)	2.00	3.00	2.00*	2.00*	1.80	2.40	1.80	1.80
O(6)	2.50	4.50	4.50*	2.50*	1.38	1.52	0.00	1.38
SU(3)	3.33	7.00	23.7*	24.7*	1.40	1.48	0.00	0.00

We first look at the effect of monopole-pairing with parameters $g_0 \in [0, 0.50]$ and $g_2 = k = 0$. Figs. 1 and 2 show the dependence of the energy levels and the B(E2) ratios on g_0. The degenerate U(5) levels, such as the $2_2 - 4_1$ doublet and the $0_3 - 3_1 - 4_2 - 6_1$ quartet, are well reproduced at $g_0 > 0.12$, and the energy and $B(E2)$ ratios are U(5)-symmetrically valued. So a large g_0 favors a spherical phase. The critical value of $g_0 \approx 0.12$ agrees well with the empirical value $g_0 \sim 20/A$ with $A = 130$ assigned in Ref. [26]. In addition, as g_0 is very small, the calculation gives approximately the feature of the SU(3) or $\overline{\mathrm{SU(3)}}$ symmetry. It indicates that a fermion system

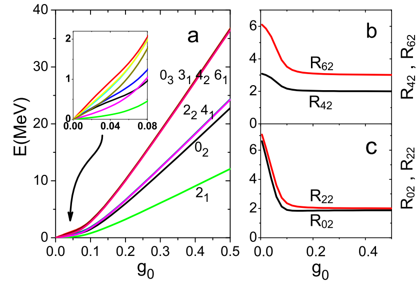

FIG. 1: The dependence of low-lying levels on g_0 when $g_2 = k = 0$.

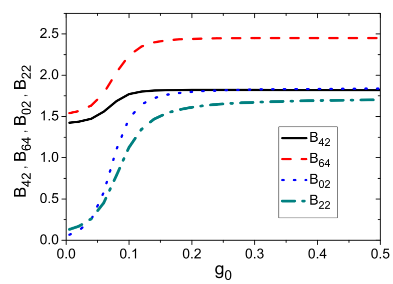

FIG. 2: The dependence of B_{42}, B_{64}, B_{02}, and B_{22} on g_0 when $g_2 = k = 0$.

with very weak monopole-pairing may appear in an axially deformed shape.

Next we examine the effects of the QQ interaction, which is known to coincide with Elliott's model [27], where the Q_2 is a quadrupole tensor in the SU(3) symmetry. However, a system may not be limited to the SU(3) phase in a more realistic model with one-body terms. So we did calculations with parameters $k \in [0, 0.50]$ and $g_0 = g_2 = 0$. The energy levels and the $B(E2)$ ratios as functions of k are shown in Figs. 3 and 4 respectively, where k is limited to below 0.20 in part (a) of Fig. 3 for a better view of the energy levels. There is clearly a special point $k \approx 0.10$, at which all the quantities defined in Table I are approximately symmetric. More specifically, this point corresponds to an O(6) phase with $R_{42} \approx R_{22} \approx 2.5$, $R_{62} \approx R_{02} \approx 4.5$, $B_{42} \approx 1.38$, $B_{64} \approx 1.52$, $B_{02} \approx 0$, and $B_{42} \approx 1.38$, in excellent agreement with the corresponding values in Table I. In addition, the degeneracy of the $2_2 - 4_1$ doublet and the $0_2 - 3_1 - 4_2 - 6_1$ quartet is clearly seen in part (a) of Fig. 3. On the other hand, at the two extremes of $k < 0.005$ and $k \geq 0.2$, the axially symmetric rotation phase prevails and the calculated energies and $B(E2)$ ratios well reproduce those of the SU(3) symmetry in Table I.

It is known that an axially symmetric rotor could have either a prolate or an oblate shape, corresponding to the SU(3) or $\overline{\mathrm{SU(3)}}$ phase in IBM-1. Since the pa-

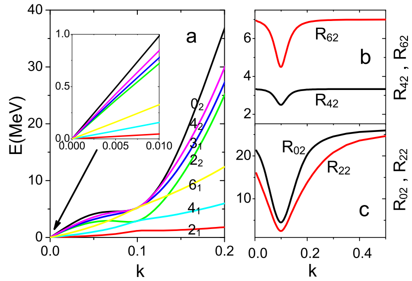

FIG. 3: The calculated energy levels as functions of k when $g_0 = g_2 = 0$.

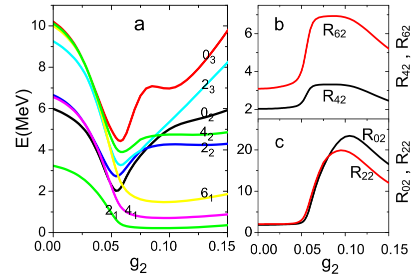

FIG. 5: The dependence of low-lying levels on g_2 when $g_0 = 0.15$ and $k = 0$.

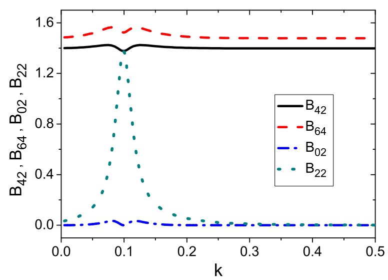

FIG. 4: The calculated $B(E2)$ ratios B_{42}, B_{64}, B_{02}, and B_{22} as functions of k when $g_0 = g_2 = 0$.

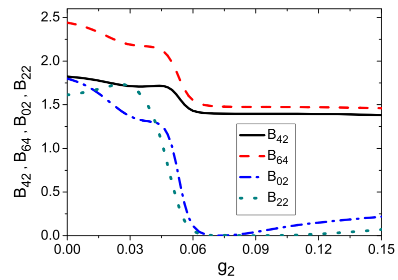

FIG. 6: The dependence of B_{42}, B_{64}, B_{02}, and B_{22} on g_2 when $g_0 = 0.15$ and $k = 0$.

rameters in Table I are the same for both phases, we have to implement other quantities to distinguish them. One parameter serving this purpose is the quadrupole moment $Q(2_1^+)$ which is negative-, zero-, and positive-valued in the SU(3), O(6), and $\overline{\mathrm{SU(3)}}$ phases respectively [15]. As the QQ interaction strength k increases from below to above 0.1, the calculated $Q(2_1^+)$ (with the effective charge normalized to 1) decreases from a positive value to zero and then to negative, which indicates an evolution of symmetry $\overline{\mathrm{SU(3)}} \rightarrow \mathrm{O(6)} \rightarrow \mathrm{SU(3)}$, namely, transitions from oblate to prolate shape with a critical point of γ-soft rotation in between. This agrees excellently with the variation behavior against the parameter χ in Ref. [15]. In addition, reexamining the effect of the g_0, we know that, as g_0 is very small, the $Q(2_1^+)$ is positive. It manifests that the fermion system with weak monopole-pairing takes an oblate shape, *i.e.*, it appears as a plateau. Such a result reproduces the recently observed density distribution of trapped cold atoms[3, 4] qualitatively.

Finally, we study how quadrupole-pairing affects the shape-phase structure, using parameters $g_2 \in [0, 0.15]$, $g_0 = 0.15$, $k = 0$. Note that $g_2 < g_0$ holds true for most realistic depictions of nuclei [13]. The calculated dependence of low-lying levels and $B(E2)$ ratios on the quadrupole-pairing strength is shown in Figs. 5 and 6 respectively. For $g_2 \in [0, 0.04]$, the system is evidently in a vibration phase. For $g_2 \in [0.07, 0.10]$, it is in an axially symmetric rotation phase, with the corresponding parameters in Table I well reproduced. Furthermore, $Q(2_1^+)$ is negative in this region. It shows that the $g_2 \in [0.07, 0.10]$ generates an axially prolate deformation (in SU(3) symmetry). Besides, the $g_2 \in [0.04, 0.07]$ induces a U(5)-SU(3) transition, and the $g_0 > 0.10$ suppresses collective motion.

The above analyses may be summarized in Table II for the correspondence between regions of interaction strengths in our microscopic model and the dynamical symmetries (*i.e.*, shape-phases) in the IBM. The Table indicates that the occurrence of an axially oblate rotor should be rare in nuclei, as it is inaccessible to most of the interactions. But it may be common in trapped cold atomic systems since the monopole-pairing there is very week. It may be noted that, calculations using different configuration parameters may result in different regions and critical values of interaction strengths corresponding to various shape-phases, but the general correspondence between regions of interaction strengths and shape-phases and the overall shape-phase structure should remain for the characteristic features of an interacting fermion system.

In conclusion, we have presented a microscopic method of interacting fermion systems and studied the dependence of their shape-phases on the strengthes of

TABLE II: The correspondence between microscopic parameter settings and shape-phases (dynamical symmetries).

	U(5)	SU(3)	O(6)	$\overline{SU}(3)$
g_0 (with $g_2 = k = 0$)	> 0.12	—	—	very small
g_2(with $g_0 = 0.15$, $k = 0$)	$[0, 0.04]$	$[0.07, 0.10]$	—	—
k (with $g_0 = g_2 = 0$)	—	≥ 0.20	0.10	≈ 0.0

basic interactions. Our study yields a complete shape-phase structure of such systems by providing an extended Casten triangle exhibiting modes of collective vibrations, axially prolate, oblate, and γ-soft rotations, corresponding to different regions of monopole-pairing, quadrupole-pairing, and QQ interaction strengthes. Specifically, strong or weak monopole-pairing leads to a vibration or an axially oblate deformation, respectively. While quadrupole-pairing and QQ interactions also induce rotations (i.e., deformations). In detail, a continuous increase of the QQ strength sees oblate, $\gamma-$soft, and prolate rotors subsequently. With a fixed strength of vibration-corresponding monopole-pairing, a suitable quadrupole-pairing strength may induce a transition from spherical to axially prolate elliptical phases. But an exceedingly large quadrupole-pairing strength suppresses collective motions. Other than nuclei, the theory should be applicable to general fermion systems such as trapped atomic fermions, where the interaction strengths may be conveniently tunable. In particular, our calculations indicate that an axially deformed shape may be induced in trapped atomic systems by either a weak monopole-pairing or a suitable quadrupole-pairing interaction, then the recently observed PIDs and nonuniform density distribution in trapped atomic systems[3, 4] could be understood as the effect of the deformation. Besides, the quadrupole-pairing could also be a trigger to a normal to LOFF phase transition in fermion systems because the D-pair is just the polarized (i.e., inhomogeneous) pair.

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